

Lessons 15-16: Related Rates

Important Formulas:

Area of a Square: $A = s^2$

Area of a Circle: $A = \pi r^2$

Circumference of a Circle: $C = 2\pi r$

Surface Area of a Cube: $A = 6s^2$

Volume of a Cube: $V = s^3$

Pythagorean Theorem: $a^2 + b^2 = c^2$

Method:

1. Sketch a picture (if applicable).
2. What rate do you want to find? What do you know?
3. Find a formula and plug in any constants.
4. Take the derivative of the formula with respect to t .
5. Plug in your known values.
6. Solve for what you want to find.
7. Answer the question.

Problems:

1. Assume that x and y are both differentiable functions of t and $x^2y = 2$. Find $\frac{dx}{dt}$ when $x = 1$ and $\frac{dy}{dt} = 5$.
2. If the radius of a circle is shrinking at 3 cm/sec, how quickly is the circumference of the circle shrinking when the radius is 4 cm?
3. If the area of a circle is increasing at π cm²/sec, how quickly is the radius of the circle changing when the radius is 3cm?
4. If each side of a cubical ice cube is decreasing at a constant rate of 0.5 cm/min,
 - (a) how quickly is the surface area of the ice cube shrinking when each side is 5cm?
 - (b) How quickly is the volume of the ice cube shrinking at that moment?
5. The volume of a cone is $V = \frac{1}{3}\pi r^2h$ where r is the radius of the base and h is the height. A melted ice cream cone is dripping at a rate of 1 cubic inch per minute. The height (altitude) of the cone is four times the diameter of the top of the cone. How quickly is the height of ice cream decreasing when there are two inches of ice cream remaining?

6. The volume of a cube is decreasing at $1 \text{ in}^3/\text{sec}$. How quickly is the side length of the cube decreasing when the side length is 4 inches?
7. The surface area of a sphere is $A = 4\pi r^2$, where r is the radius of the sphere. The radius is decreasing at a rate of 2 cm/sec . How quickly is the surface area changing when the radius is 3 cm?
8. The base of a 10-ft ladder that is leaning against a wall is pushed towards the wall. When the base is 6 ft from the wall and moving at the rate of 2 ft/sec , how fast is the top of the ladder sliding up the wall?
9. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation is changing when the angle is: (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
10. Two airplanes depart the Purdue Airport. One leaves at noon heading due east at 550 miles per hour and the other leaves at 12:30pm heading due north at 600 miles per hour. How quickly is the distance between them changing at 1:30pm?
11. An airplane flying at an altitude of 10 miles passes directly over a radar antenna. When the airplane is 15 miles away, the radar detects that the distance is changing at a rate of 250 miles per hour. What is the speed of the airplane?
12. A boat is pulled into a dock by means of a winch 15 feet above the deck of the boat. The winch pulls in rope at a rate of 5 feet per second. Determine the speed of the boat when there is 39 feet of rope out.
13. In softball, the distance between each base is 60 feet. A player is running from second base to third base at a speed of 16 feet per second. Find the rate at which the distance from home plate is changing when the player is 20 feet from second base.